

Optical Bloch oscillations in periodic structures with metamaterials

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We predict that optical Bloch oscillations can be observed in layered structures with left-handed metamaterials and zero average refractive index where the layer thickness varies linearly across the structure. We demonstrate a new type of the Bloch oscillations associated with coupled surface waves excited at the interfaces between the layers with left-handed material and conventional dielectric.

Electron oscillations in the presence of a constant electric field were predicted by Bloch in 1928 [1]. Such Bloch oscillations become possible due to beating of the localized eigenmodes of the structure corresponding to the equidistant eigenstates of the spectrum known as the Wannier-Stark ladder [2]. Experimental verification of the theory was impossible at that time, since dephasing time of electrons in crystals is shorter than the period of the electron Bloch oscillation. Later, electron Bloch oscillations were observed in semiconductor superlattices [3] for which the period was reduced due to a small miniband width in the artificial structure.

Dephasing processes for electromagnetic waves are negligible making the observation of the optical Bloch oscillations in photonic systems much easier. The first experimental observation of optical Bloch oscillations was reported in Ref. [4] for linearly chirped Bragg gratings. Later, several studies reported the observation of optical Bloch oscillations in various structures [5, 6, 7, 8, 9].

Recent experimental realization of left-handed materials [10] has opened up many unique opportunities to explore novel effects in the structures with negative refractive index. In this Letter we study, for the first time to our knowledge, optical Bloch oscillations in one-dimensional layered structures containing alternating layers of left-handed and conventional dielectric slabs. We choose the material parameters in such a way that the average refractive index \bar{n} across pair of the neighboring layers vanishes, thus fulfilling the condition for the existence of a novel type of the specific zero- \bar{n} bandgap [11, 12]. We change the layer thickness linearly in the structure and observe an optical analogue of the Wannier-Stark ladder in the eigenmode spectrum, and the corresponding Bloch oscillations in the resonant transmission bands. We reveal that in such structures the Bloch oscillations can be observed in three different regimes. Compared to the photonic Bloch oscillations in conventional dielectric structures, the metamaterial structures can support a novel type of the Bloch oscillations associated with coupling of surface waves at the interfaces between left-handed and dielectric layers.

We study a one-dimensional layered structure shown schematically in Fig. 1, where the slabs with the width b_i are made of metamaterial being separated by a dielectric slab with the width a_i . Variation of the refractive index

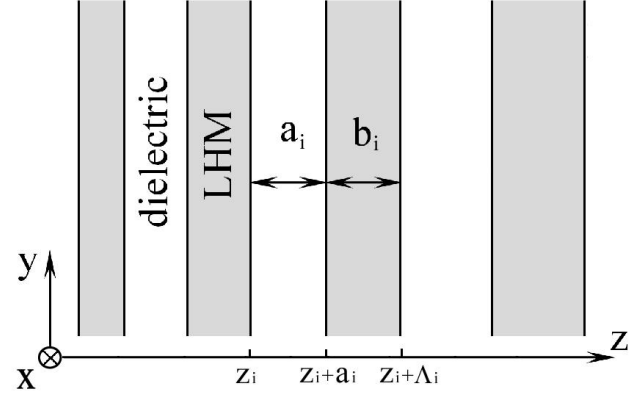


FIG. 1: Schematic of linearly chirped one-dimensional photonic crystal with alternating layers of left-handed metamaterial and dielectric.

in the i -th pair of layers can be described as follows:

$$n(z) = \begin{cases} n_r = \sqrt{\epsilon_r \mu_r} & z \in (z_i, z_i + a_i) \\ n_l = -\sqrt{\epsilon_l \mu_l} & z \in (z_i + a_i, z_i + \Lambda_i) \end{cases} \quad (1)$$

where n_l and n_r are the refractive indices of metamaterial and dielectric, respectively.

We consider TE-polarized waves with the electric field having one component $\mathbf{E} = (E_x, 0, 0)$, and waves propagating in the plane (y, z) . In this case, the field distribution can be described by the Helmholtz equation:

$$\Delta_2 E_x(y, z) + n^2(z) E_x(y, z) - \frac{1}{\mu} \frac{d\mu(z)}{dz} \frac{\partial E_x(y, z)}{\partial z} = 0, \quad (2)$$

where Δ_2 is the two-dimensional Laplacian, and the coordinates are normalized to c/ω . Firstly we consider periodic structure. Electric field in an infinite one-dimensional *periodic structure* can be represented as a superposition of Bloch eigenmodes [13], with the electric field envelopes $U(z + \Lambda) = U(z)$, where Λ is the structure period. The dispersion relation for the Bloch waves is found by the transfer matrix method [13],

$$2 \cos(K_B \Lambda) = 2 \cos(k_{zr} a) \cos(k_{zl} b) - \left(\frac{k_{zl} \mu_r}{k_{zr} \mu_l} + \frac{k_{zr} \mu_l}{k_{zl} \mu_r} \right) \sin(k_{zr} a) \sin(k_{zl} b), \quad (3)$$

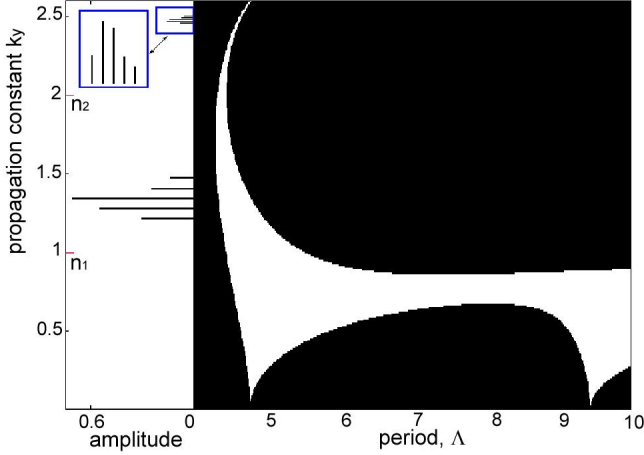


FIG. 2: (Color online) Bandgap diagram for the TE-polarized waves. Black and white areas correspond to gaps and bands, respectively. Two spectra of the excited Bloch oscillations are shown on the left. The inset shows a magnified part of the spectrum.

where K_B is the Bloch wavenumber, $k_{z,l,r} = \mp \sqrt{n_{l,r}^2 - k_y^2}$ and k_y is the normalized propagation constant along the y axis. According to this relation an infinite stack containing metamaterials exhibits a non-resonant gap for $\bar{n} \equiv \Lambda^{-1} \int_0^\Lambda n(z) dz = 0$, where \bar{n} is an average refractive index of the structure, i.e. $n_r a = |n_l b|$. This condition is easy to fulfil for negative refractive index materials.

The bandgap diagram of the layered structure is shown in Fig. 2 for the parameter plane (Λ, k_y) . Here we assume that dielectric is vacuum, $\epsilon_r = \mu_r = 1$, and that it is two times thicker than the second layer, $a/b = 2$. We choose the parameters of the left-handed media as follows: $\epsilon_r = -5$ and $\mu_r = -0.8$. This set of parameters allows surface waves to exist at the interfaces between metamaterial and vacuum [12]. As follows from Fig. 2, for the zero \bar{n} structure the bandgap spectrum differs substantially from the case of conventional periodic structures made of conventional dielectrics [13]. Stack with the average zero refractive index possesses a complete gap with the transmission resonances [11, 12] when the optical path of the wave in either layer of the period coincides with a half of the wavelength in the corresponding medium. Thus for the normal incidence ($k_y = 0$) transmission is observed only when $n_r a = n_l b = \pi m$, where m is integer. For the slabs of equal thickness the regions of the transmission resonances in (Λ, k_y) plane degenerate into infinitely thin lines.

We study the propagation of electromagnetic waves in such a layered structure with zero average refractive index in each pair of layers, when the thickness of layers is chirped linearly, i.e. $\Lambda_q = \Lambda_0 + q\delta\Lambda$, where integer q numbers the layers. We are looking for localized solutions in the structure, and for numerical simulations we

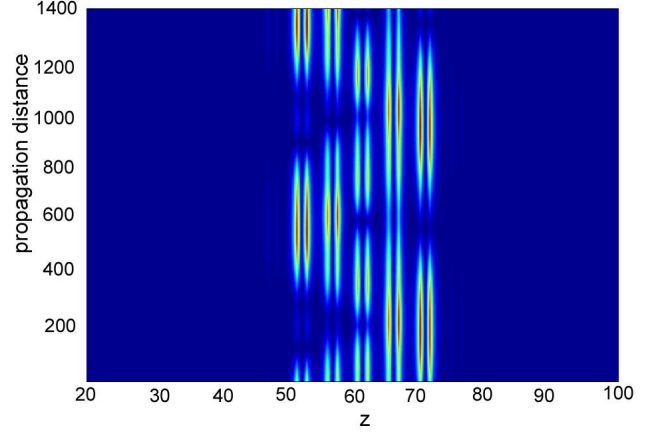


FIG. 3: (Color online) Field distribution in the case of surface-wave-assisted Bloch oscillations. The Wannier-Stark ladder appears for the propagation constants centered around $k_{y0} = 2.47$, normalized period is $L_y = 820$.

consider a finite stack of layers with perfect metal boundary conditions, $E(z=0) = E(z=L) = 0$, where L is the total length of the structure. We assume the Gaussian field distribution in the plane $y=0$ across the layers, and in order to find the electromagnetic field distribution in the whole stack we look for its eigenmodes by solving the Helmholtz equation (2). Then we decompose the initial field distribution using the basis of eigenmodes and find the solution in the whole structure.

To find the eigenmodes of Eq. (2) we employ the following discretization scheme [14]:

$$\frac{2}{\mu_{m-1}^{-1} + \mu_{m+1}^{-1}} \left[\frac{U_{m+1} - U_m}{\mu_{m+1}} - \frac{U_m - U_{m-1}}{\mu_{m-1}} \right] \frac{1}{h^2} + \left(\frac{\epsilon_{m-1} + \epsilon_{m+1}}{\mu_{m-1}^{-1} + \mu_{m+1}^{-1}} \right) U_m = k_y^2 U_m, \quad (4)$$

where $z_m = mh$ are the mesh points with the discretization step h and $E_x(y, z) = U(z)e^{-jk_y y}$. Such a discretization scheme provides an algorithm convergence [14], and it avoids excitation of spurious modes in the structure.

In metamaterials the energy flow $\int [\mathbf{E} \times \mathbf{H}] dz$ can be negative, i.e. the energy can propagate in the opposite direction to the propagation constant \mathbf{k}_y [15]. Consequently, we determine the direction of the energy flow of each eigenmode and choose the sign of the propagation constant such that the energy flows in the positive y -direction. Decomposition of the initial condition in the plane $y=0$ in the eigenmode basis is made using the least squares method.

To find propagation constants (values of k_y) which lead to the Bloch oscillations, we analyze the spectrum of eigenvalues of this layered structure. The Bloch oscillations are expected to appear where the spectrum of eigenmodes is equidistant. Practically for all gradients of a linear ramp we observe several sets of equidistant states. The equidistant eigenvalues of k_y correspond to

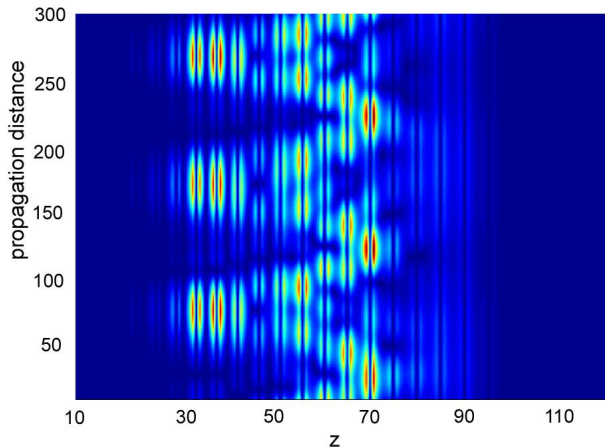


FIG. 4: (Color online) Field distribution for the case of guided waves. The Wannier-Stark ladder appears for the propagation constants centered around $k_{y0} = 1.34$, period is $L_y \cong 100$.

a spatial optical equivalent of the Wannier-Stark ladder which is associated with the Bloch oscillations.

Spectrum of k_y can be divided into three different regions. First, when $k_y < n_r < |n_l|$, electromagnetic waves propagate in both left- and right-handed materials. In the second region, $n_r < k_y < |n_l|$, waves propagate in metamaterial only being evanescent in the vacuum layers. In this regime, our structure can be considered as an array of coupled left-handed waveguides. When $k_y > |n_l| > |n_r|$, only surface waves may propagate along the interfaces separating different materials.

We find that the Bloch oscillations can be observed in all three regimes of the wave propagation when the corresponding set of equidistant propagation constants is excited. We consider a stack containing 36 pairs of metamaterial and dielectric slabs and the normalized period Λ varying from 3.7 to 6. First, we excite the eigenstates corresponding to the regime of surface waves with the center of the spectrum at $k_{y0} = 2.47$. Figure 3 presents the intensity distribution for the electric field which shows clearly spatially periodic oscillations of the beam position in the structure. The corresponding spectrum of eigenstates is shown on the left side of Fig. 2. We note that the beam reconstructs its shape after each period of oscillations. The field is highly confined to the interfaces between metamaterial and vacuum, demonstrating that such Bloch oscillations exist due to interaction of surface waves in the structure. The distance between Wannier-Stark eigenstates Δk_y defines the period of oscillations, $L_y = 2\pi/\Delta k_y$. For this case, we find $L_y = 820$, and this agrees well with Fig. 3.

Bloch oscillations of the beam with the spectrum cor-

responding to the coupled waveguide regime, $n_r < k_y < |n_l|$, are shown in Fig. 4. The equidistant spectrum of eigenstates corresponding to the Wannier-Stark ladder is also shown in Fig. 2 (top, left). We notice that

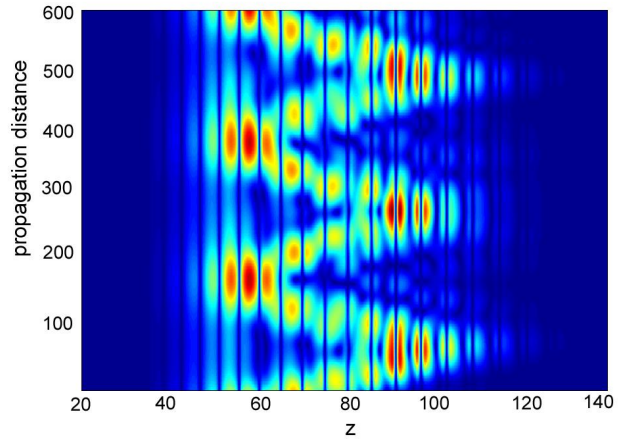


FIG. 5: (Color online) Field distribution for the case when waves propagate in both type of materials. The Wannier-Stark ladder appears with average propagation constant $k_{y0} = 0.8$, period of oscillations is $L_y = 210$.

oscillations are strongly anharmonic, but they are still periodic with the period defined well by the relation $L_y = 2\pi/\Delta k_y$, which is less than the period of Bloch oscillations associated with surface waves.

The regime of Bloch oscillations corresponding to the waves propagating in both media can be found in a different structure with wider transmission resonance. We analyse that the structure consisting of 36 periods and where the normalized period varies linearly from 2.5 to 7.5 (corresponding to the period change gradient $\delta\Lambda = 0.14$). Ratio of the layer thicknesses in each period is the same as in the previous calculations. We choose $\varepsilon = -3.6$ and $\mu = -1.11$, preserving the zero average refractive index of the structure. The calculated field distribution in this case is shown in Fig. 5, and the center of the equidistant spectrum appears at $k_{y0} \sim 0.8$.

In conclusion, we have studied the propagation of electromagnetic waves in layered structures with left-handed metamaterials, and have demonstrated that linearly chirped structures with zero average refractive index can support novel types of Bloch oscillations. We have demonstrated that the excitation spectra are equidistant, manifesting a similarity with the optical Wannier-Stark ladder. Using numerical simulations, we have demonstrated three different types of the Bloch oscillations, and we have revealed the existence of unusual oscillations associated with coupled surface waves.

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